I.E.S./I.S.S. Examination, 2026

वियोज्य *DETACHABLE*

STATISTICS

Paper IV

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **FOURTEEN** questions divided under **SEVEN** Sections.

Candidate has to choose any **TWO** Sections and attempt the questions therein. All the Sections carry equal marks.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Diagrams/Figures, wherever required, shall be drawn in the space provided for answering the question itself. F-table and Graph paper can be found at the end of both sections in the booklet.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer (QCA) Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

SECTION 'A'

(Operations Research and Reliability)

1. Answer **all** of the following : 10×5=50

1. (a) A paint company produces both interior and exterior paints using two raw materials M_1 and M_2. The following table provides the basic data of the problem :

	Tons of raw material per ton of		Maximum daily availability (tons)
	Exterior paint	Interior paint	
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit per ton (in thousand Rs.)	50	40	

The daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. Also the maximum daily demand for interior paint is 2 tons. Determine the optimal product mix. 10

1. (b) Neon lights on the campus of a university are replaced at the rate of 100 units per day. It costs Rs. 10,000 to initiate a purchase order. A neon light kept in storage is estimated to cost about Rs. 2/day. The lead time between placing an order and receiving the order is 12 days. Determine optimal inventory policy for ordering the neon lights. Also, calculate cycle length and reorder point of the order. 10

1. (c) There are five jobs to be performed by employees in a Departmental Store. The time (in hours) that each employee takes to perform each job is given in the following effective matrix :

		Employees				
		I	II	III	IV	V
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How the jobs should be assigned, one per employee, so as to minimize the total manhours ? 10

1. (d) Two jobs are to be performed on five machines A, B, C, D and E. Processing times (in hours) are given below :

Machines						
Job 1	Sequence	A	B	C	D	E
	Processing Time	3	4	2	6	2
Job 2	Sequence	B	C	A	D	E
	Processing Time	5	4	3	2	6

Use graphical method to obtain minimum elapsed time and idle times for the two jobs. 10

1. (e) Define the terms :
IFR, IFRA, NBU, NBUE, DMRL 10

2. Answer any **two** from the following : 25×2=50

(a) Consider the following table summarizing the details of a project involving 14 activities :

Activity	*Immediate predecessor(s)*	*Duration (in months)*
A	–	2
B	–	6
C	–	4
D	B	3
E	A	6
F	A	8
G	B	3
H	C, D	7
I	C, D	2
J	E	5
K	F, G, H	4
L	F, G, H	3
M	I	13
N	J, K	7

Draw the network diagram and identify the critical path of the project. 25

2. (b) (i) Write the basic characteristics of queueing model (M/M/I) : (FCFS/∞/∞). 5

(ii) A company has received a contract to supply gravel to three new construction projects located in three towns A, B, C from 3 gravel pits located in three towns X, Y and Z. The delivery cost from each pit to each project site of truckloads is given below :

		Project Location			
Pits		A	B	C	Supply
	X	4	8	8	76
	Y	16	24	16	82
	Z	8	16	24	77
	Demand	72	102	41	

Find an optimum schedule of transportation so as to minimize the total cost of transportation. 20

2. (c) (i) Discuss the replacement policy for items whose running cost increases with time and value of money remains constant during a period. 10

(ii) Suppose that a sample of 12 items were put on test and the test was terminated at the 8th failure time, and the lifetimes follow an exponential distribution with mean $\theta > 0$. The failure times are, in hours

30, 57, 145, 167, 320, 440, 505, 675

(I) obtain maximum likelihood estimate of mean lifetime

(II) construct 95% confidence interval for θ

[Given $\chi^2_{(16,\ 0\cdot975)} = 28\cdot845$; $\chi^2_{(16,\ 0\cdot025)} = 6\cdot908$] 15

2. (d) (i) Obtain an optimal solution using simplex method to the following linear programming problem :

Maximize $z = 3x_1 + 2x_2$

Subject to

$$
\begin{aligned}
-x_1 + 2x_2 &\leqslant 4\\
3x_1 + 2x_2 &\leqslant 14\\
x_1 - x_2 &\leqslant 3\\
x_1, x_2 &\geqslant 0
\end{aligned}
$$

Also find an alternative optimal solution if it exists. 15

(ii) When do we apply the Rule of Dominance in solving a game problem ? Solve the game whose payoff matrix is given below : 10

		Player B		
Player A		I	II	III
	I	30	40	–80
	II	0	15	–20
	III	90	20	50

SECTION 'B'

(Demography and Vital Statistics)

3. Answer **all** of the following : 10×5=50

3. (a) Define Central Mortality Rate (m_x) and Force of Mortality (μ_x). 5

Establish the relation

$$\mu_{x+\frac{1}{2}} = m_x$$

stating the necessary assumptions involved. 5

3. (b) (i) Define crude death rate with its limitations. 4

(ii) Explain standardized death rates including the methods of their computation. 6

3. (c) Ascertain whether Gompertz's or Makeham's formula for graduation is suitable for the following data :

Age x	35	40	45	50	55	60
$l(x)$	89785	86237	82377	78018	72895	66666

10

3. (d) Derive an algebraic expression relating the probability of a person surviving between age x and $x+1$, p_x to the force of mortality, μ_x. 10

3. (e) Fill in the blanks in a portion of life table given below :

Age (x)	l_x	d_x	p_x	q_x	L_x	T_x	ρ_x°
4	90000	500	?	?	?	5060200	?
5	?	300	?	?	?	?	?

10

4. Answer any **two** from the following : 25×2=50

(a) What is vital statistics ? State the uses of vital statistics. Explain registration method and census method of obtaining vital statistics. 5+5+15=25

(b) The sex wise distribution of population and number of births with survival rates of a town in 2022 are given below :

Age group	*Population*		*Births*		*Survival rate* $(n\pi x)$
	Male	*Female*	*Male*	*Female*	
15-19	6432	6210	65	64	0·92
20-24	5318	5430	120	115	0·90
25-29	4470	4580	110	105	0·87
30-34	3960	3970	80	78	0·86
35-39	3580	3600	60	65	0·84
40-44	3140	3050	15	18	0·83
45-49	2890	2705	4	3	0·81

Compute GFR, ASFR, TFR, GRR, NRR. 25

(c) In usual notations as in a life table, given $l_{80} = 16000$ and

Age (x)	80	81	82	83	84	85	86
d_x	5000	3000	2500	2000	1000	1000	500

(i) Compute the values of l_x and q_x for $x = 81, 82, ..., 86$. 10

(ii) The ages of three persons A, B and C are 81, 82 and 83 respectively. Find the probabilities.

(1) that A, B and C will be alive in two years' time 5

(2) that one at least of the three will be alive in two years' time 5

(3) that exactly one of them will be alive in two years' time 5

(d) What is internal and international migration ? 10

Discuss various classifications of internal and international migration. 15

SECTION 'C'

(Survival Analysis and Clinical Trials)

5. Answer **all** of the following : 10×5=50

(a) For gamma lifetime model $f(x, b, p) = \frac{b^p e^{-bx} x^{p-1}}{\sqrt{p}}$, $b > 0, p > 0, x > 0$, show that hazard function is an increasing function for $p > 1$ and decreasing function for $p < 1$. 10

(b) One often hears that the death rate of a person that smokes is, at each age, twice that of a non-smoker. Does it mean that a non-smoker has twice the probability of surviving a given number of years as does a smoker of the same age ? Justify your answer. 10

(c) Define hazard function, cumulative hazard function and mean residual life function. If T is a continuous non-negative random variable with cumulative hazard function $H(T)$ then show that $H(T)$ follows standard exponential distribution. 10

(d) What is multicenter trial ? Why are multicenter trials conducted ? 10

(e) Consider two groups of survival data with hazards $\lambda_1(t)$, $\lambda_2(t)$ and survivor functions $s_1(t)$, $s_2(t)$ respectively.

(i) One of the assumptions for Cox model is proportional hazard, what is really meant by "proportional hazard" ? 5

(ii) Assuming the two hazard functions are not the same, examine the connection between the crossings of the two hazard functions and the crossing of the two survivor functions. 5

6. Answer any **two** from the following : 2×25=50

(a) The survival times of two groups of breast cancer patients who had surgical treatment are given below :

Survival group (in months) : $3, 7^+, 9, 9, 11^+, 16$
Chemotherapy group (in months) : $8, 9, 10^+, 12^+, 18, 23^+$

Apply logrank test for comparing survival distributions of the two groups at 5% level of significance using exact method.
[Given $\chi_1^2(0{\cdot}05) = 3{\cdot}841$] 25

(b) Explain time censoring and number censoring (type-II censoring). State the likelihood functions in each censoring scheme. Assume the life time follows exponential distribution with failure rate $\frac{1}{\lambda}$, $\lambda > 0$.

Derive the maximum likelihood estimate of survival function at time $t = 700$ based on the following censored data :

$t_1 = 80,\ t_2 = 95,\ t_3 = 105,\ t_4 = 180,\ t_5 = 270,\ t_6 = 330,\ t_7 = 670,\ t_8 = 800,\ t_i \geqslant 1000,$ $i = 9,\ 10,\ \dots,\ 20.$

Using the (i) actual failure time/survival time 13

(ii) number of failures observed only without considering the actual failure time. 12

(c) A study is made on the impact of regular exercise and gender on the risk of developing heart diseases amongst 55-75 year olds. A sample of people was followed from 0 if female, 1 if male, the exact age of 55 years until they either develop heart diseases or turn 75 years, whichever comes first. The Cox PH model was used for this study with the two covariates being defined as

$$z_1\ (\text{Gender}) = \begin{cases} 0, & \text{if female} \\ 1, & \text{if male} \end{cases};\qquad z_2\ (\text{exercise rate}) = \begin{cases} 0, & \text{if rare} \\ 1, & \text{if regular} \end{cases}$$

The model results were as follows :

Model fitted		*Maximum log likelihood*
1. Null model	:	–1190
2. Gender only	:	–1177
3. Gender and Exercise	:	–1170
4. Gender, Exercise and Interaction	:	–1166 ; (Interaction = Gender * Exercise)

Covariate		*Parameter fitted*
Gender	:	$\beta_1 = 0{\cdot}25$
Exercise	:	$\beta_2 = -0{\cdot}35$
Gender * Exercise	:	$\beta_0 = -0{\cdot}45$

(i) Give two reasons why the Cox PH model is suitable in this data analysis.

(ii) Perform a statistical test to show that the interaction term is significant in the model (Take $\alpha = 0{\cdot}05$).

(iii) Give the hazard functions for a male who does not exercise regularly; a female who exercises regularly; a female who rarely exercises and a male who exercises regularly.

(iv) Identify the baseline hazard for this model.

(v) Interpret your results in (iv) with reference to the baseline hazard and the hazard function for males who rarely exercise.
[Given : $\chi^2_{(1,\ 0\cdot05)} = 3\cdot841$] 2+8+8+2+5=25

(d) What is randomization in a clinical trial ? How should the randomization code be determined ? Discuss some common randomization methods. 5+4+16=25

SECTION 'D'

7. Answer **all** of the following : 10×5=50

(a) Why do we make use of Statistical Quality Control ? Write the advantages when a process is working in a state of statistical control. 10

(b) Explain the statistical reasoning for using 3-σ limits in statistical quality control. 10

(c) A Metropolitan Transit system uses the number of written passenger complaints per day as a measure of its service quality. For 10 days, the number of complaints received are given below :

Day (sample) no.	1	2	3	4	5	6	7	8	9	10
No. of complaints/day	4	8	2	0	3	9	10	0	6	4

Obtain the three control limits for the above data. 10

(d) Explain the differences between CUSUM and Shewhart Control Charts in terms of their uses and methodology. 10

(e) Describe Double-Sampling Inspection plan and discuss its advantages. 10

8. Answer any **two** from the following : 25×2=50

(a) A machine is set to deliver packets of a given weight. Ten samples of size 5 each were recorded and the recorded data is produced below :

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{x}$)	15	17	15	18	17	14	18	15	17	16
Range (R)	7	7	4	9	8	7	12	4	11	5

Can the process be regarded under control ?
(Given conversion factors for $n = 5$, $A_2 = 0\cdot58$, $D_3 = 0$, $D_4 = 2\cdot115$) 25

(b) Explain the statistical basis and construction of *p* and *np* charts. How is the choice between *p* and *np* charts made ? 25

(c) (i) It has been decided to sample 100 items at random from each large batch and to reject the batch if more than 2 defectives are found. The acceptable quality level is 1% and unacceptable quality level is 5%. Find the producer's and consumers risks. 10

(ii) Define a single sampling plan.
Consider a single sampling plan :
Lot size (N) = 2000, Sample size (n) = 50 and
acceptance number of defectives (c) = 2.
Find the probability of accepting the lot. 15

(d) Hourly concentration (X_i) data collected from a chemical process is given below :

Hour	X_i	*Hour*	X_i
1	5·50	11	6·75
2	4·50	12	3·25
3	5·25	13	5·25
4	6·0	14	5·0
5	5·25	15	4·5
6	3·50	16	6·50
7	5·75	17	7·20
8	6·25	18	6·80
9	4·50	19	6·75
10	5·0	20	6·50

If the target mean $(\mu_0) = 5{\cdot}0$, $n = 1$ and $\sigma = 1$, use CUSUM Control Chart to detect the shift to $\mu_1 = 6{\cdot}0$ for $K = 0{\cdot}5$. 25

SECTION 'E'

(Multivariate Analysis)

9. Answer **all** of the following : 10×5=50

(a) Given $\underset{\sim}{X} \sim N_3\,(\underset{\sim}{\mu}, \Sigma)$, where

$$\underset{\sim}{\mu} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} 8 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 3 \end{pmatrix},$$

(i) find the regression function of X_1 on X_2 and X_3, and

(ii) compute the conditional variance of X_1 given X_2 and X_3. 10

(b) Show that $\underset{\sim}{X} = (X_1, X_2, ..., X_p)'$ has p-variate normal distribution if and only if every linear combination $(l_1 X_1 + l_2 X_2 + ... + l_p X_p)$ of $\underset{\sim}{X}$ follows a univariate normal distribution. 10

(c) Let $T_p^2 = n\underset{\sim}{Y}'A^{-1}\underset{\sim}{Y}$, where $\underset{\sim}{Y} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ and $A \sim W_p(n, \Sigma)$ which is independent of $\underset{\sim}{Y}$. Show that $T_p^2 \geqslant T_K^2$ for $K \leqslant p$. 10

(d) Let there be two populations π_1 and π_2. It is known that about 30% of all objects belong to π_2 and

C $(2|1)$: cost incurred when a π_1 observation is incorrectly classified as π_2 observation = 15;

C $(1|2)$: cost incurred when a π_2 observation is incorrectly classified as π_1 observation = 10.

Suppose the two density functions $f_1(x)$ and $f_2(x)$ (corresponding to π_1 and π_2) are evaluated at a new observation x_0 and $f_1(x_0) = 0{\cdot}32$, $f_2(x_0) = 0{\cdot}56$. Can the new observation be classified as coming from π_1 or π_2 ? 10

(e) Let $\underset{\sim}{X} = (X_1, X_2, X_3)'$ has the correlation matrix R given by

$$R = \begin{bmatrix} 1 & \frac{1}{4\sqrt{2}} & 0 \\ \frac{1}{4\sqrt{2}} & 1 & \frac{1}{4\sqrt{2}} \\ 0 & \frac{1}{4\sqrt{2}} & 1 \end{bmatrix}$$

Obtain the first two principal components and the percentage of population variance explained by the first two principal components. 10

10. Answer any **two** from the following : 25×2=50

(a) (i) If $\underset{\sim}{X}$ is a random p-vector distributed as $N_p(\underset{\sim}{\mu}, \Sigma)$, then obtain the distribution of $\underset{\sim}{X}'\Sigma^{-1}\underset{\sim}{X}$ and specify its parameters. 15

(ii) If $A \sim W_p(n, \Sigma)$, then prove that $CAC' \sim W_q(n, C\Sigma C')$, where C is a $(q \times p)$ matrix of rank $q \leqslant p$. 10

(b) Define sample generalised variance based on a random sample $\underset{\sim}{X}_\alpha (\alpha = 1, 2, ..., N)$ of size N drawn from $N_p(\underset{\sim}{\mu}, \Sigma)$ and obtain its distribution. Also find an expression for its h^{th} moment $(h = 1, 2, ...)$.

If $\underset{\sim}{Z}_1$, $\underset{\sim}{Z}_2$ and $\underset{\sim}{Z}_3$ are independently and identically distributed as $N_2(\underset{\sim}{0}, \Sigma)$ with

$\Sigma = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, then obtain $E\left|\sum_{\alpha=1}^{3} \underset{\sim}{Z}_\alpha \underset{\sim}{Z}'_\alpha\right|$. 25

(c) Let $X_1, X_2, ..., X_p$ represent measurements or characteristics on one member of a twin pair and $X_{p+1}, X_{p+2}, ..., X_{2p}$ represent the same measurements on the other member. Assuming that

$$\underset{\sim}{X} = (X_1, X_2, ..., X_{2p})' \sim N_{2p}(\underset{\sim}{\mu}, \Sigma),$$

with Σ unknown, develop a suitable test for testing the hypothesis of equality of measurements of the twins. 25

(d) Define canonical correlations and canonical variates and obtain the characteristic equation they satisfy. Hence, or otherwise, show that multiple correlation and simple correlation are special cases of canonical correlation. 25

SECTION 'F'

(Design and Analysis of Experiments)

11. Answer **all** of the following : 10×5=50

(a) In an RBD there are only two blocks. Let K be the number of treatments and $\bar{x}_j$, $j = 1, 2$ the average yield of j^{th} block. Show that the between block sum of squares can be expressed as $\frac{K}{2}(\bar{x}_1 - \bar{x}_2)^2$ and write the ANOVA table. 10

(b) Explain the two basic ways of testing hypothesis involving contrasts of K parameters. Explain orthogonal contrasts and their use. 10

(c) Suppose you have υ varieties compared in V^2 plots. How will you carry out the experiment under each of the following situations ?

(i) there is no fertility difference among the V^2 plots.

(ii) the fertility changes along two perpendicular direction.

Write the appropriate ANOVA table for each case. 5+5

(d) State the advantages of a factorial experiment over a simple experiment. Explain Yates' method of computing factorial effect totals. 10

(e) Define the linear model for a split-plot design with two factors replicated *r* times. Write the ANOVA table. Give an example of the design. 10

12. Answer any **two** from the following : 25×2=50

(a) In the table given below are the yields of 6 varieties in a 4 replicate experiment for which one observation under treatment 2 in block 2 is missing. Estimate the missing observation and analyse the data :

Blocks	*Treatments*					
	1	2	3	4	5	6
1	18·5	15·7	16·2	14·1	13·0	13·6
2	11·7	–	12·9	14·4	16·9	12·5
3	15·4	16·6	15·5	20·3	18·4	21·5
4	16·5	18·6	12·7	15·7	16·5	18·0

(Given : $F_{3,14}(0{\cdot}05) = 3{\cdot}34$, $F_{5,14}(0{\cdot}05) = 2{\cdot}96$) 25

(b) Diet affects weight gain. We wish to compare nine diets : these diets are the factor level combinations of protein source (beef, pork and grain) and number of calories (low, medium and high). There are test animals nine in number that were randomly assigned to the nine diets one animal per diet. The responses (weight gain) are :

Source	*Calories*		
	Low	*Medium*	*High*
Beef	76·0	86·8	101·8
Pork	83·3	89·5	98·2
Grain	83·8	83·5	86·2

Using an appropriate linear model analyse the data and give the ANOVA table. (Given : $F_{2,4}(0{\cdot}05) = 6{\cdot}44$) 25

(c) Define the analysis of covariance model of CRD with one concomitant variable and explain its statistical analysis. 25

(d) In designing a battery for use in a device three possible plate materials are tested at three temperature levels. The following table gives the life (in hours) of the plate materials :

Plate material type	*Temperature* (°F)		
	15	70	125
1	130, 155 74, 180	34, 40 80, 75	70, 58
2	159, 126	136, 115	45
3	138, 160	150, 139	96

Considering the given data is the proportional data, assess the effect of temperature on the plate material type and present the ANOVA table.
(Given : $F_{4,11}(0{\cdot}05) \simeq 3{\cdot}36$, $F_{2,11}(0{\cdot}05) = 3{\cdot}98$). 25

SECTION 'G'

(Computing with C and R)

13. Answer **all** of the following : 10×5=50

(a) Summarise the rules for type conversion in C when neither operand is unsigned. Explain conditional operator with a suitable example. 10

(b) Given a matrix $A_{m \times n}$ write a C function to find the product of $A_{m \times n}$ and its transpose and to print the result. 10

(c) Describe two different approaches to updating a data file. Write illustrative programs one each. 10

(d) Write R code to create a data frame with name, age and gender of 10 individuals.

(i) Extract age and gender of the 4th, 8th and 1st individual in this order.

(ii) Add three new records to the data frame created above. 10

(e) Write R code to find the median of the observations x_i ($i = 1, 2, ..., 10$) without using the median function. 10

14. Answer any **two** from the following : 25×2=50

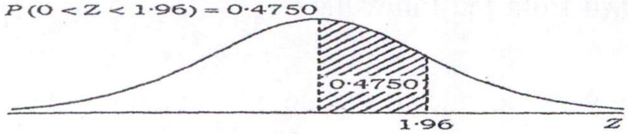
(a) Given the observed frequency distribution with the variate values x_i ($i = 0, 1, 2, ..., 8$) and the corresponding frequencies f_i ($i = 0, 1, 2, ..., 8$) write a C program to fit a binomial distribution and test for its goodness of fit and to print the result. 25

(b) Given class, section, name, date of birth, roll number and marks secured in four subjects for a group of 30 students write a C program to find the average mark secured by each student and to print roll number and average mark of each student. Make use of structure variables within the program. 25

(c) Write a C program to test for the independence of attributes in a given $m \times n$ contingency table and to print the result. 25

(d) (i) For a given observed frequency distribution with variate values x_i ($i = 0, 1, 2, ..., 10$) and the corresponding frequencies f_i ($i = 0, 1, 2, ..., 10$), write *R* code to find mean and variance without using *R* functions and compare them. Also print your comment. 15

(ii) Write R code to generate a random sample of size 15 from $N(22, \sigma = 1 \cdot 5)$ and another random sample of the same size from $N(13, \sigma = 2)$ and to test for the equality of the means of the two populations at 1% level of significance and to print the model values of the samples. 10



The Normal Distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Percentage Points of the *F* Distribution

$F_{0{\cdot}05,\, v_1,\, v_2}$

Degrees of Freedom for the Denominator (v_2)

v_2 \ v_1	Degrees of Freedom for the Numerator (v_1)																	
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00